Chapter 8: SURDS

Surds are square roots of numbers which don't simplify into a whole (or rational) number: e.g. $\sqrt{2} \approx 1.414213...$ but it is more accurate to leave it as a surd: $\sqrt{2}$

General rules

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\sqrt{a} \times \sqrt{a} = \sqrt{a^2} = a$

But you cannot do:

$\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

These are NOT equal

$(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = \sqrt{a^2} + \sqrt{ab} - \sqrt{ab} - \sqrt{b^2} = a - b$

Simplifying Surds

Find the largest square numbers and simplify as far as possible

Worked Examples

$\sqrt{18} = \sqrt{2 \times 9} = \sqrt{2} \times \sqrt{9} = \sqrt{2} \times 3 = 3\sqrt{2}$ Careful - this is "3 times the square root of 2" NOT "the cube root of 2"

Rationalising the Denominator

This is a fancy way of saying getting rid of the surd on the bottom of a fraction. We multiply the fraction by the denominator (or the denominator with the sign swapped)

Worked Examples

1. *Rationalise* $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$ we multiply by $\frac{a}{a}$ which is the same as multiplying by 1, which means we don't fundamentally change the fraction.
2. *Rationalise* $\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{10}$
3. *Rationalise* $\frac{1}{\sqrt{5} + \sqrt{2}} = \frac{1}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{1 \times (\sqrt{5} - \sqrt{2})}{(\sqrt{5} + \sqrt{2}) \times (\sqrt{5} - \sqrt{2})}$

   $= \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5^2} + \sqrt{2} \times \sqrt{5} - \sqrt{2} \times \sqrt{5} - \sqrt{2^2})} = \frac{\sqrt{5} - \sqrt{2}}{5 + \sqrt{10} - \sqrt{10} - 2} = \frac{\sqrt{5} - \sqrt{2}}{3}$
4. *Rationalise* $\frac{\sqrt{2}}{3\sqrt{2} - 1} = \frac{\sqrt{2}}{3\sqrt{2} - 1} \times \frac{3\sqrt{2} + 1}{3\sqrt{2} + 1} = \frac{\sqrt{2} \times (3\sqrt{2} + 1)}{(3\sqrt{2} - 1) \times (3\sqrt{2} + 1)}$

   $= \frac{3\sqrt{2^2} + \sqrt{2}}{(3^2\sqrt{2^2} + 3\sqrt{2} - 3\sqrt{2} - 1^2)} = \frac{3 \times 2 + \sqrt{2}}{9 \times 2 - 1} = \frac{6 + \sqrt{2}}{17}$

Exercise A:
Simplify the surds

1) $\sqrt{12}$
2) $\sqrt{125}$
3) $\sqrt{48}$
4) $\sqrt{72}$
5) $\sqrt{27}$

Exercise B:
Expand and simplify

1) $\sqrt{2}\left(3+\sqrt{5}\right)$
2) $\sqrt{6}\left(\sqrt{2}+\sqrt{8}\right)$
3) $4(\sqrt{5}+3)$
4) $(2+\sqrt{3})(1+\sqrt{3})$
5) $(3-\sqrt{5})(3-2\sqrt{5})$
6) $(2+\sqrt{5})(2+\sqrt{3})$
7) $(1-\sqrt{2})(1+\sqrt{3})$
8) $(8-\sqrt{2})(8+\sqrt{2})$
9) $(\sqrt{3}+\sqrt{5})(\sqrt{3}+\sqrt{5})$

Exercise C:
Rewrite the following expressions with rational denominators

1) $\dfrac{3}{\sqrt{5}}$
2) $\dfrac{4}{\sqrt{8}}$
3) $\dfrac{9}{\sqrt{48}}$
4) $\dfrac{\sqrt{2}+1}{2}$
5) $\dfrac{\sqrt{3}-1}{\sqrt{5}}$
6) $-\dfrac{4}{3\sqrt{2}}$
7) $\dfrac{1}{\sqrt{3}-1}$
8) $\dfrac{4}{\sqrt{6}-2}$
9) $\dfrac{7}{\sqrt{7}-2}$
10) $\dfrac{-3}{\sqrt{5}+1}$
11) $\dfrac{\sqrt{3}-1}{\sqrt{5}}$
12) $\dfrac{\sqrt{5}-1}{\sqrt{5}+3}$