Chapter 4: FACTORISING

Common factors

We can factorise some expressions by taking out a common factor.

Example 1: Factorise $12x - 30$

Solution: 6 is a common factor to both 12 and 30. We can therefore factorise by taking 6 outside a bracket:

$$12x - 30 = 6(2x - 5)$$

Example 2: Factorise $6x^2 - 2xy$

Solution: 2 is a common factor to both 6 and 2. Both terms also contain an x.
So we factorise by taking $2x$ outside a bracket.

$$6x^2 - 2xy = 2x(3x - y)$$

Example 3: Factorise $9x^3y^2 - 18x^2y$

Solution: 9 is a common factor to both 9 and 18.
The highest power of x that is present in both expressions is x^2.
There is also a y present in both parts.
So we factorise by taking $9x^2y$ outside a bracket:

$$9x^3y^2 - 18x^2y = 9x^2y(xy - 2)$$

Example 4: Factorise $3x(2x - 1) - 4(2x - 1)$

Solution: There is a common bracket as a factor.
So we factorise by taking $(2x - 1)$ out as a factor.
The expression factorises to $(2x - 1)(3x - 4)$

Exercise A

Factorise each of the following

1) $3x + xy$

2) $4x^2 - 2xy$

3) $pq^2 - p^2q$

4) $3pq - 9q^2$

5) $2x^3 - 6x^2$

6) $8a^5b^2 - 12a^3b^4$

7) $5y(y - 1) + 3(y - 1)$

Factorising quadratics

Simple quadratics: Factorising quadratics of the form $x^2 + bx + c$
The method is:
Step 1: Form two brackets $(x \; \ldots \;)(x \; \ldots \;)$
Step 2: Find two numbers that multiply to give c and add to make b. These two numbers get written at the other end of the brackets.

Example 1: Factorise $x^2 - 9x - 10$.

Solution: We need to find two numbers that multiply to make -10 and add to make -9. These numbers are -10 and 1.
Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

General quadratics: Factorising quadratics of the form $ax^2 + bx + c$
The method is:
Step 1: Find two numbers that multiply together to make ac and add to make b.
Step 2: Split up the bx term using the numbers found in step 1.
Step 3: Factorise the front and back pair of expressions as fully as possible.
Step 4: There should be a common bracket. Take this out as a common factor.

Example 2: Factorise $6x^2 + x - 12$.

Solution: We need to find two numbers that multiply to make $6 \times -12 = -72$ and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} + \underbrace{9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Difference of two squares: Factorising quadratics of the form $x^2 - a^2$

Remember that $x^2 - a^2 = (x + a)(x - a)$.

Therefore: $x^2 - 9 = x^2 - 3^2 = (x + 3)(x - 3)$
$16x^2 - 25 = (2x)^2 - 5^2 = (2x + 5)(2x - 5)$

Also notice that: $2x^2 - 8 = 2(x^2 - 4) = 2(x + 4)(x - 4)$
and $3x^3 - 48xy^2 = 3x(x^2 - 16y^2) = 3x(x + 4y)(x - 4y)$

Factorising by pairing
We can factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing:

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)
$= (2x + y)(x - 1)$

If you need **more help** with factorising, you can download a booklet from this website:
http://www.mathcentre.ac.uk/resources/workbooks/mathcentre/web-factorisingquadratics.pdf

Exercise B

Factorise

1) $x^2 - x - 6$

2) $x^2 + 6x - 16$

3) $2x^2 + 5x + 2$

4) $2x^2 - 3x$ (factorise by taking out a common factor)

5) $3x^2 + 5x - 2$

6) $2y^2 + 17y + 21$

7) $7y^2 - 10y + 3$

8) $10x^2 + 5x - 30$

9) $4x^2 - 25$

10) $x^2 - 3x - xy + 3y^2$

11) $4x^2 - 12x + 8$

12) $16m^2 - 81n^2$

13) $4y^3 - 9a^2 y$

14) $8(x+1)^2 - 2(x+1) - 10$